CHAPTER 7

# *De incessu animalium* 9
## *Aristotle's Mathematical Kinesiology: The Case of Bending*

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### Introduction

The focus of *IA* 9 is bending's role in animal locomotion. The claim that initiates the chapter concerns the relationship between bending and rest. Aristotle argues that if nothing were at rest, neither bending nor straightening could occur. But the majority of the chapter concerns the relationship between bending and locomotion. Aristotle defends the claim that there could be no walking, swimming, flying, or any other variety of animal locomotion without bending.

That locomotion involves bending may seem obvious on perceptual grounds. But the import of Aristotle's discussion resides more in the form of argument he employs in support of the conclusion – namely, a narrowly geometrical argument – than in the conclusion itself. It is important, nevertheless, to note that if Aristotle is able to establish both of the chapter's principal claims, they imply, by a hypothetical syllogism, a central Aristotelian tenet, namely, that there could be no animal locomotion if nothing were at rest.

I will break up the chapter into four sections. Section 1 comprises Aristotle's definition of bending and his argument that bending requires a point at rest. In section 2, Aristotle provides a geometrical argument for the claim that animal locomotion requires bending. To properly interpret this argument, we must broach several methodological issues. Most notably, we will discuss the role that principles of pure mathematics can play in explanations of physical phenomena. The role geometry plays in this argument is quite unlike the appeals to geometry one finds elsewhere in the biological works. I will argue that Aristotle's explanation belongs neither to pure geometry nor to observational biological science, but rather to an applied mathematical science I call *mathematical kinesiology*. Section 3 discusses bending at joints other than the knee and provides a second geometrical depiction of animal locomotion. Section 4 extends

Aristotle's findings to animals that crawl, swim, fly, and "ooze." I aim to better understand how Aristotle conceives bending, to explore and evaluate the ways he appeals to bending in his explanations of animal locomotion, and to situate the methodology on display within the subtly interrelated network of sciences Aristotle countenances.

## Bending and Rest [*IA* 9, 708b21–26]

Aristotle defines bending (κάμψις) as "the change from what is straight to what is curved or angled" (708b22–23). Movements in the contrary directions, from what is curved or angled back to what is straight, are not instances of bending, but rather of straightening. Elsewhere, Aristotle defines bending as "a change to the convex or the concave without a change in the length" (*Mete.* IV 9, 386a1–2). This alternative definition differs in two ways from the definition offered in the *IA*, but is, despite these differences, ultimately compatible with it. First, the *Meteorology's* definition mentions only changes from what is straight to what is curved (i.e., convex or concave). It is clear that the *IA* considers changes to what is angled instances of bending; though snakes, caterpillars, birds, and many fish move, either entirely or in part, by curving their limbs and bodies, most animals, especially those that walk, move by bending their limbs at joints. But the purpose of *Mete.* IV 8–9 is to catalog, explain, and provide examples of the passive capacities that differentiate homoeomerous natural bodies. Changes from what is straight to what is angled do not appear elsewhere in this discussion, so, given how comprehensive Aristotle intends the list of passive capacities to be, it is safe to conclude that Aristotle considers such changes instances of bending.[1] Second, the *Meteorology*'s definition restricts bending to changes that do not involve a change in the length of that which bends. Though Aristotle's definition in *IA* does not mention this feature of bending, the claim that the lengths of



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[1] Aristotle's list comprises eighteen pairs of passive capacities: (i) capable or incapable of solidification, (ii) meltable or unmeltable, (iii) softenable or unsoftenable by heat, (iv) softenable or unsoftenable by water, (v) bendable or unbendable, (vi) breakable or unbreakable, (vii) capable or incapable of fragmentation, (viii) impressible or unimpressible, (ix) mouldable or unmouldable, (x) squeezable or unsqueezable, (xi) tractile or non-tractile, (xii) malleable or non-malleable, (xiii) fissile or non-fissile, (xiv) cuttable or uncuttable, (xv) viscous or friable, (xvi) compressible or incompressible, (xvii) combustible or incombustible, and (xviii) capable or incapable of giving off fumes (*Mete*. IV 8, 385a14–18).

limbs neither increase nor decrease during locomotion plays an important role in the arguments Aristotle goes on to make. So Aristotle's considered view, which he employs in *IA* and is compatible with his other discussions, is that bending is a change from what is straight to either what is curved (i.e., concave or convex) or what is angled without a change in the length of that which bends.

Aristotle's argument that bending and straightening require that something be at rest is quite brief.

> That if nothing were at rest no bending or straightening could occur, is evident from what follows. For bending is the change from what is straight to what is curved or angled, straightening is the change from either of these to what is straight. In all such changes, the bending or straightening must necessarily be relative to one point. (*IA* 9, 708b21–26)

Beyond the definition of bending, the only substantive claim Aristotle makes is that both bending and straightening are necessarily relative to one point. It is this single point that is at rest.

It is an open question what this resting point is. There are at least two ways to bend, say, a straight pipe, into an arc. Either someone can hold one of the endpoints fixed and exert pressure on the other endpoint, or one can exert pressure on both of the endpoints simultaneously. If the arc that results from the second kind of bending is a circular arc, there are two options for what the point at rest could be (Figure 7.1*a*). It could be either the midpoint of the pipe or the theoretical center point, i.e., the focus, which determines the arc by being equidistant from every point the arc comprises. But the theoretical option seems incorrect for two reasons. First, there is no single point that determines other varieties of arc – elliptical arcs are determined by a pair of foci and parabolic arcs are determined by a point, its focus, and a line, its directrix (Figures 7.1*b* and *c*). Second, if the result of bending is neither convex nor concave, but is an angle, the point at rest is clearly the angle's vertex (Figure 7.1*d*).

In the discussion that follows, Aristotle treats the bending of limbs in footed animals to be the principal case. And when a limb, say, a leg, bends, the point at rest is in the limb's central joint, viz. the knee. That this is so is evident from Aristotle's other discussions of the movements of limbs. Aristotle is clear that,

> if one of the parts of an animal be moved, another must be at rest, and this is the purpose of their joints; animals use joints like a center, and the whole member, in which the joint is, becomes both one and two, both

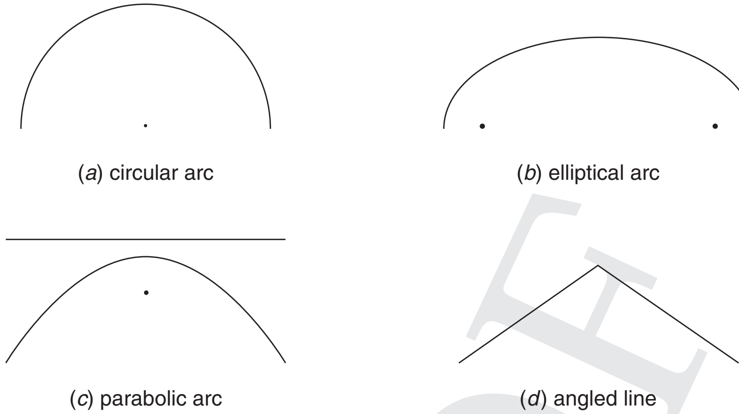



Figure 7.1 Possible resting points for a bending

> straight and bent, changing potentially and actually by reason of the joint. (*MA* 1, 698a17–21, trans. Farquharson)[2]

If bending that results in an arc is analogous, the point at rest would be the arc's apex or vertex.

### Aristotle's Geometrical Argument [*IA* 9, 708b26–709a7]

Having shown that bending requires a point at rest, Aristotle goes on to offer a remarkable geometrical argument for the claim that animal locomotion requires bending.

> Moreover, without bending there could not be walking or swimming or flying. The reason is that, since footed animals stand and take their weight alternately on one or the other of their opposite legs, as one leg strides forward the other must necessarily be bent. For the opposite legs are naturally of equal length, and the one that is under the weight must be a kind of perpendicular <line> at right angles to the ground. When, then, one leg strides forward, it becomes the hypotenuse of a right-angled triangle. Its

[2] Strictly speaking, the point at rest is not the joint as a whole, but a point within the joint. Aristotle says that "something initiates motion instrumentally when the starting point and the end point are the same, for instance in a hinge joint; for here the convex is the end point and the concave the starting point (for which reason the latter is at rest and the former is moved), and though differing in account, they are inseparable in magnitude. For all things are moved by pushing and pulling; consequently, it is necessary, just as in the case of a circle, for something to remain fixed and for the motion to begin from there" (*DA* III 10, 433b21–27, trans. Shields, slightly modified; cf. *Meta.* VII 16, 1040b9–14). For a discussion of the several types of joint, see *PA* II 9, 654b15–23; *HA* I 15, 493b30–31; and *De spir.* 7, 484b22–26.

> square then is equal to the square on the other side together with the square on the base. But since the legs are equal, the one at rest must bend either at the knee or, in any kneeless animal that walks, at some other joint. (*IA* 9, 708b26–709a4)

This argument warrants an extended treatment. I will analyze the argument, highlight how unique this sort of geometrical argument is within Aristotle's biological works, and situate the argument within Aristotle's more general accounts of geometrical, physical, and biological explanation.

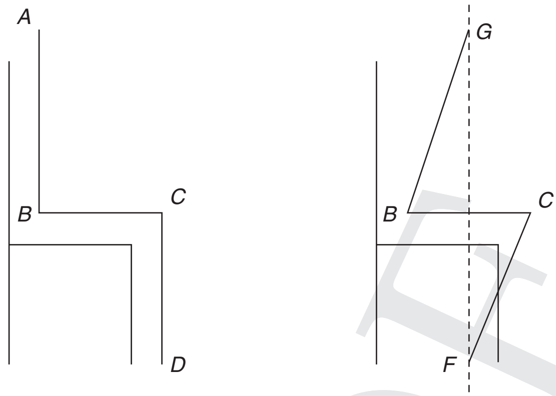
### *The Argument*

Though Aristotle's stated conclusion is that walking, swimming, and flying require bending, the argument only discusses footed animals that walk. Aristotle does ultimately establish that swimming and flying (as well as other varieties of animal locomotion) require bending, but he does not extend this argument's results to other ways of progressing until 709a24.

The initial premise that footed animals alternate their weight upon their opposite legs is more subtle than it may initially appear. We can see what it means to take one's weight on a leg by looking at a passage in the *Mechanica*. *Mech.* 30 concerns what one must do to rise from a seated position.[3] It is impossible to stand up from a chair if one keeps both one's lower legs and one's back perpendicular to the ground (just try!). In this position, one's weight, that is, one's center of gravity, is not upon one's feet; if the chair were removed one would fall backwards onto the ground (Figure 7.2*a*). To rise, one must form two acute angles by leaning one's head forward and moving one's feet inward toward the body. Only then will the seated individual be "at right angles to the ground" where this means that he will "have his head in the same line as his feet" (*Mech.* 30, 857b29–31). That is, it is only when a line that is roughly perpendicular to the ground can be drawn through the head and the feet that one will be in a position to take one's weight on one's feet and stand (Figure 7.2*b*).[4]

[3] Though scholars disagree about *Mechanica*'s authenticity, the explanation present in chapter 30 contains the very same technical terminology that Aristotle employs in the geometrical argument under consideration and, as we will see, its meaning in the former clearly sheds light on the latter usage.

[4] "Why is it that when people rise from a sitting position, they always do so by making an acute angle between the thigh and the lower leg and between the chest and the thigh, otherwise they cannot rise? […] Let A be the head, AB the line of the chest, BC the thigh, and CD the lower leg. Then AB, the line of the chest, is at right angles to the thigh, and the thigh at right angles to the lower leg, when a man is seated in this way. In this position, then, a man cannot rise; but to do so




(*a*) weight not on feet (*b*) weight on feet

Figure 7.2 Rising from a seated position

In his geometrical argument, Aristotle assumes that walkers take their entire weight on one leg at a time. Consistent with the discussion of rising from a seated position, whichever leg "is under the weight must be a kind of perpendicular <line> at right angles to the ground" (708b31–32). That is, the walker's center of gravity will remain over the trailing leg. If the leg that extends forward remains straight, the walker's legs will form a right triangle with the ground. The lead leg will be the hypotenuse of this triangle and the trail leg upon which the walker's weight rests will be the side of the triangle perpendicular to the ground. So if the length of one's leading leg is five units and the walker steps forward three units, the length from the walker's hip to the ground must be four units (Figure 7.3*b*). This is simply an application of the Pythagorean theorem to the triangle the walker's legs form. But a typical walker's legs are of equal length, and when one walks, the length of one's legs neither increases nor decreases. So the only way for the distance from the hip to the ground to be four units is for the trailing leg to bend at the knee (Figure 7.3*c*).

Aristotle's geometrical argument does not depend directly upon observable phenomena. But he does appeal to such phenomena to support his conclusion. Though incomplete in all presently available manuscripts,

he must bend the leg and place the feet at a point under the head. This will be the case if CD be moved to CF [and if AB be moved to GB], and the result will be that he can rise immediately, and he will have his head and his feet in the same straight line; and CF will form an acute angle with BC [and GB will form an acute angle with BC]" (*Mech*. 30, 857b21–858a2, trans. Forster with additions).

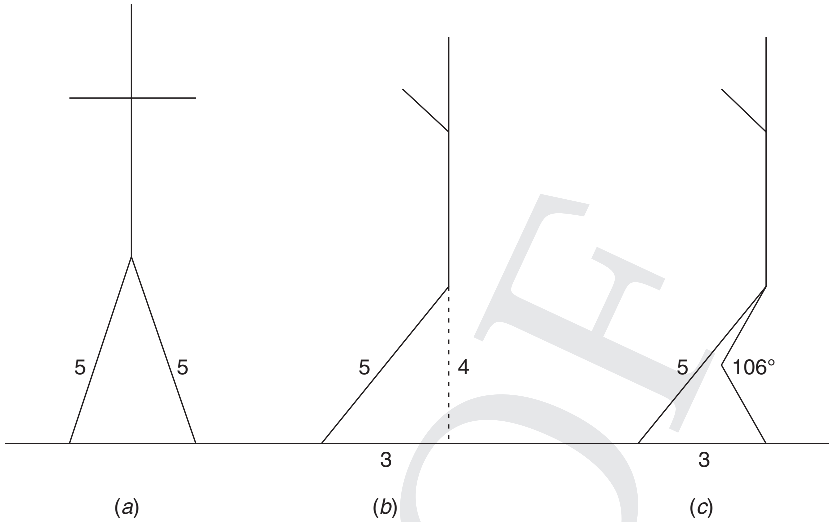



Figure 7.3 Bending the trail leg when walking

Michael of Ephesus provides a full characterization of the evidence to which Aristotle appeals (Michael of Ephesus, *In IA* 154.18–155.7) and his characterization allows us to fill in what is missing in Aristotle's text as follows:

> This is shown by the following fact: if a human being were to walk on the ground along a wall <having attached to his head a reed dipped in ink> the line described <by the reed> would not be straight but zigzag, because it would go lower when the human being bends and higher when it stands and raises itself. (*IA* 9, 709a4–7)

When a walker's legs are farthest apart, the head is lower than the position it occupies when the walker's legs are together. As the walker alternates between these orientations, the head descends and rises.[5]

If walkers move as Aristotle's geometrical argument depicts, the line it describes would be zigzag (Figure 7.4*a*). In actual fact, the line it describes is a series of curves (Figure 7.4*b*).

[5] Michael of Ephesus supplies even more evidence: "If someone were to walk along a wall as high as his eye-level, and someone else the same height were to stand on the other side of the wall, he would not see continuously the top of the head of the one walking, but when the walker extended his leg, he would not see the top of his head because of his becoming shorter, but when the advanced foot is pulled up, he would see the head because it is lifted and gets up to its height again" (Michael of Ephesus, *In IA* 155.1–7, trans. Preus).


Figure 7.4 The position of the head when walking

### *Geometrical Argumentation in the Biological Works*

Aristotle's natural philosophy is replete with appeals to mathematical theorems, including those of geometry.[6] But almost all appeals to geometry within the biological works belong to one of three classes: (i) analogy, (ii) illustration, and (iii) non-geometrical description of observable morphology.

Analogical appeals to geometry do not involve direct applications of geometrical claims to observable phenomena. Instead, they aim to show that the relationships in which various biologically relevant items stand resemble, in important respects, those that obtain among geometrical *relata*. For example, in *DA* II 3, Aristotle argues that the way in which a lower soul is present in the soul of a comparatively sophisticated organism, e.g., the way in which a nutritive soul is present in an animal's perceptual soul, is analogous to the way in which a simple figure is present in a comparatively complex figure, for instance the way in which a triangle is present in a quadrangle (*DA* II 3, 414b20–415a1). But souls do not possess any other features of figures and even the one respect in which souls and figures are analogous is not realized identically in their respective subjects. Moreover, in analogical arguments of this sort, the conclusions are not themselves established geometrically.

Illustrative appeals to geometry do involve the application of geometry to observable phenomena. But these applications are heuristic and can be eliminated without altering the non-geometrical arguments they serve to illuminate. For example, in *MA* 1, Aristotle argues that when a limb bends at a joint, one point within the joint is moved and another point within the joint remains at rest. He elucidates this conclusion by saying that this is "just as would happen if, on a diameter, AD were to remain at rest and B moved so as to bring about AC" (*MA* 1, 698a22–24; Figure 7.5).

This geometrical illustration is helpful, but we can ultimately dispense with it, because the non-geometrical arguments Aristotle provides before

[6] See Hussey 1991: 213–242 for a discussion of several examples.

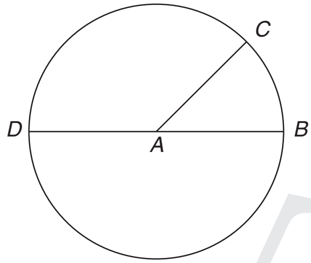



Figure 7.5 The geometrical argument in *MA*

he states the claim illuminated by the illustration sufficiently establish it as a conclusion.[7]

Appeals to geometry within descriptions of observable morphology involve genuinely explanatory applications of geometrical concepts. But these uses do not involve properly geometrical arguments. For example, in *PA* IV 5, Aristotle concludes that sea urchins must have five "ova," five teeth, and five stomachs, and the fact that their bodies are spherical is an ineliminable premise in his argument. But the explanatory role that the sphericity of a sea urchin's body plays in Aristotle's argument does not reside in any narrowly geometrical features of spheres. Instead, their sphericity matters because Aristotle maintains that the distribution of objects on spherical bodies must be balanced (*PA* IV 5, 680b4–681a3).

The argument in *IA* 9 does not fit comfortably into any of these familiar patterns. Its conclusion, like the conclusions of the arguments we have just discussed, concerns the proper subject matter of biology insofar as the argument purports to establish a general truth about the physiology and activity of footed animals. But Aristotle's argument about bending differs from the others in being a genuine explanation that conforms to

[7] That Aristotle views this geometrical example heuristically is further confirmed by his insistence that he is not considering the illustrative circle's center to be realized physically (and thereby "potentially and actually now one, now divided"), but is only considering it mathematically (and thereby "indivisible in every respect") (*MA* 1, 698a26–28; cf. *Mem.* 1, 450a1–6). *MA* contains two other illustrative appeals to geometry, in chapters 9 and 11. The best candidate for a genuinely geometrical argument occurs in chapter 7. Aristotle argues that small qualitative changes due to perception can have large consequences for bodily movement because the parts of the body involved in motion stand to the perceptual organ, the heart, as the circumference of a circle stands to its center, and "it is not hard to see that a small change occurring at the center makes great and numerous changes at the circumference, just as by shifting the rudder a hair's breadth you get a wide deviation at the prow" (*MA* 7, 701b24–28, trans. Farquharson; cf. *Phys.* VIII 4, 254b28–30, *Mech.* 848a10–19). But though Aristotle could employ this theorem concerning circular motion in a non-illustrative way, it is not obvious that its presence in this passage (and its application to ships) is anything but illustrative or analogical.

the methodological prescriptions of the *Posterior Analytics*.[8] In addition, though the definition of bending is not a principle of geometry, the argument's major premise, which employs the Pythagorean theorem, unequivocally is. The argument's explanatory force depends upon the truth of this narrowly geometrical principle and its presence is, in contrast to the other appeals to geometry we have discussed, ineliminable.

Before turning to a more thorough exploration of the positive role geometrical principles play in Aristotle's argument about bending, we should mention a related respect in which this argument differs from the other geometrical appeals one finds in the biological works. That is, the argument's premises are barely tethered to observable phenomena. The only physiological assumptions Aristotle makes are that walkers (i) do not progress by moving all of their limbs at once; (ii) progress in a way that does not involve their falling down; (iii) have limbs that do not change their length when they progress; and (iv) have limbs of equal length. The first pair of assumptions are arguably included in the account of walking, the third is included in the account of bending, and the last could be dispensed with, though doing so would require a more complex argument for the same conclusion.

Moreover, though Aristotle goes on to cite observable phenomena in support of the argument's conclusion, the argument itself depicts a gait that is wildly at variance with what anyone can easily observe. It involves no bending at the ankle or the hip. The lead leg remains completely straight. And it requires that one squat over one's trail leg (Figure 7.6).

In his notes to Michael of Ephesus' commentary, Anthony Preus remarks upon the gulf between the gait Aristotle's geometrical argument depicts and the observable facts. He says, "it is more than a little odd that Aristotle argues this way, since the facts of walking are not that difficult to observe, and in fact the Greek artists represent walking, running, and other gaits in men and animals (horses, dogs, and so on) with

[8] The demonstration is as follows.

[P1] Length of perpendicular side is $\sqrt{n2 - m2}$ *belongs to* Right triangle constructed upon perpendicular line, *a*, of length *n* with hypotenuse of length *n*, parallel side of length *m*, and perpendicular side a line segment of *a*.

[P2] Right triangle constructed upon perpendicular line, *a*, of length *n* with hypotenuse of length *n*, parallel side of length *m*, and perpendicular side a line segment of *a belongs to* The legs of a footed-animal with limbs of length *n* progressing a distance of length *m* by walking.

[C1] Length of perpendicular side is $\sqrt{n2 - m2}$ *belongs to* The legs of a footed-animal with limbs of length *n* progressing a distance of length *m* by walking.

Given Aristotle's definition of bending, his conclusion follows trivially from C1.

[C2] Bending *belongs to* The legs of a footed-animal with limbs of length *n* progressing a distance of length *m* by walking.

Figure 7.6 Walking like Aristotle



considerable accuracy" (Preus 1981: 64). Are we to believe that Aristotle is unaware of what walking actually looks like?

It seems clear, given how obvious it is that actual walkers do not look the way Aristotle's argument depicts, that Aristotle is not attempting to capture the observable manner in which walkers progress. Presumably this divergence is a feature, not a bug, of the kind of geometrical argument Aristotle gives. But what, precisely, is Aristotle doing in this argument and how are we to understand geometrical arguments for biological conclusions more generally?

### *Geometrical Arguments for Biological Conclusions*

According to Aristotle, the sciences are autonomous. That is, the principles of each science concern the *per se* attributes of the objects that belong to their proprietary domains, e.g., arithmetic's subject genus is number and geometry's subject genus is spatial magnitude. Aristotle maintains a proscription against arguments whose premises and conclusions concern objects that belong to the subject genera of distinct sciences.[9] So one cannot use the principles of one science, say arithmetic, to prove claims about the proper objects of another science, say, geometry. If

[9] *APo* I 7. A nice discussion of Aristotle's injunction against such "kind-crossing" is Hankinson 2005: 23–54.

this proscription were completely general, it would exclude the very possibility of geometrical arguments for biological conclusions.

But Aristotle allows for an important class of exceptions. If the subject genus of one science "comes under another" (θάτερον ὑπὸ θάτερον), then the former science is said to be subordinate to the latter. In such cases, it is possible (and often necessary) to use the superordinate science's principles as premises in the subordinate science's arguments. For example, mathematical optics is subordinate to geometry, mechanics is subordinate to stereometry (i.e., solid geometry), and mathematical harmonics is subordinate to arithmetic.[10] In each of these cases, claims whose proper home is pure mathematics play explanatory roles in applied fields of inquiry.

The applied mathematical sciences that are subordinate to the several fields of pure mathematics also have observational sciences subordinate to them. For example, observational optics (which Aristotle says includes iridology – "the study of the rainbow") is subordinate to mathematical optics, and acoustical harmonics is subordinate to mathematical harmonics.[11] The purpose of these observational sciences is to collect the facts that the applied mathematical sciences purport to explain. So Aristotle seems to accept that some of the sciences are organized into tripartite hierarchies.[12] In such hierarchies, applied mathematical science occupies a position intermediate between entirely mathematical inquiries and entirely observational inquiries.

But there is a sense in which all three of a given hierarchy's sciences study the same objects. Pure mathematics studies the geometrical properties that natural bodies possess, but does so in a way that prescinds entirely from their natural realization. Aristotle says that "natural bodies have planes, volumes, lines, and points," and makes it clear that, in

[10] See *APo* I 7, 75b13–20; I 9, 76a16–25; I 13, 78b37–79a16; and *Meta.* XIII 3, 1078a5–21. Cf. footnote 17 for some reservations about the subordination of mechanics to stereometry.

[11] *APo* I 13, 78b35–79a13. Interestingly, when Aristotle actually discusses rainbows and other phenomena that result from the reflection of optical rays, his arguments are paradigmatic instances of applied mathematical science. He uses geometrical principles to explain the observable, physically realized, geometrical features of optical phenomena, e.g., that a halo is always a circle or a segment thereof and that a rainbow's arc is never greater than a semi-circle (*Mete.* III 3, 372b34–373a19; III 5, 375b16–377a11). Johnson 2009: 163–186 offers a thorough analysis of these arguments and, on Johnson's interpretation, Aristotle's argument for the perfect circularity of lunar and solar halos is a geometrical argument for a meteorological conclusion that mirrors, in many important respects, this essay's interpretation of the way in which the argument under consideration is a geometrical argument for a biological conclusion.

[12] The claim that subordinate sciences are arranged triadically is not entirely uncontroversial. Hankinson, Lennox, and McKirahan discuss several difficulties that attend this interpretation (Hankinson 2005: 47–50, Lennox 1986: 42–44, and McKirahan 1978: 213–215).

addition to the student of nature, "the mathematician also studies these things, but not *as* they are limits of natural bodies, nor does he study accidents *as* accidents of such bodies."[13] Aristotle summarizes the relationship between pure mathematics and applied mathematical science succinctly when he asserts that "geometry studies natural lines, but not *as* natural, whereas optics studies mathematical lines, but *as* natural, not *as* mathematical."[14]

Both applied mathematical science and observational science study the same properties of natural bodies that pure mathematics studies, but unlike pure mathematics, they study them *as* natural. But observational science does not employ mathematical principles in its explanations and is thereby unable to establish *why* the geometrical facts it determines to hold of natural bodies are as it observes them to be. When it comes to naturally realized geometrical properties, "it is for the observational scientists to know the fact (τὸ μὲν ὅτι) and for the [applied] mathematical scientists to know the reason why (τὸ δὲ διότι)."[15]

So applied mathematical science focuses on the mathematical properties that we perceive to be realized *per se* in natural bodies and considers them both *as* the mathematical properties they are and *as* the properties of the bodies that possess them. Insofar as the properties are mathematical, one can apply a relevant class of general mathematical principles to them. Insofar as these properties are naturally realized, the application of these mathematical principles can explain observable facts about the objects that possess them.[16]

Given this framework, where are we to situate Aristotle's geometrical argument in *IA* 9? It appears to be a clear instance of applied mathematical science. The argument's object is the triangularity of a walker's legs, but it does not consider it in abstraction; the argument considers the triangularity only insofar as it is realized naturally in a hypothetical walker. Moreover, the argument applies a general principle of geometry, the Pythagorean theorem, and it is this principle that explains various facts about the legs of walkers. The observationally-oriented biologist or

[13] *Phys.* II 2, 193b24–25 and 31–34.

[14] *Phys.* II 2, 194a9–11; cf. *Meta.* XIII 3, 1078a2–4. Lennox captures this dual character when he notes that in the explanations of applied mathematical sciences "[t]he middle term picks out the description of the natural object in virtue of which it has a certain mathematical property; that property is a *per se* property of a natural kind qua being a mathematical kind" (Lennox 1986: 41).

[15] *APo* I 13, 79a3–4; cf. *APr* I 30, 46a19–21.

[16] This is, of course, only the barest sketch of Aristotle's account of the applied mathematical sciences and of scientific subordination more generally. Distelzweig 2013: 85–105, Hankinson 2005: 23–34, Johnson 2015: 163–186, Lennox 1986: 29–51, and McKirahan 1978: 197–220 include especially rich discussions of these and related matters.

student of physiology can tell us that a walker's legs have certain mathematical properties, but they cannot use the presence of these mathematical properties to explain any other facts they observe to be true of walkers. The facts Aristotle's geometrical argument explains hold of all walkers' legs, but they do not hold because they are properties of legs. These facts hold because the legs of walkers, when they progress, are triangular. Only someone engaged in the relevant applied geometrical science will be in a position to explain these facts properly.

Let us call this applied mathematical science *mathematical kinesiology*.[17] The arguments of mathematical kinesiology employ principles of pure geometry to explain facts that have the naturally realized geometrical properties of progressing animals as their cause. These arguments needn't depict the manner in which walkers *typically* progress. Aristotle is not attempting to describe how walking *must* be realized physiologically. Rather, the purpose of Aristotle's argument is to place constraints on any physiological realization of walking. He proceeds as he does because it allows him to employ readily recognized theorems about right triangles. But his result can be extended easily to gaits that involve the movement of the walker's center of gravity to a position not directly over his trailing foot, to gaits that involve bending at the ankle and the hip, to gaits in which the lead leg bends, and even to the gaits of animals without knees.

[17] It is unclear how mathematical kinesiology is related to the other sciences Aristotle recognizes. Given that Aristotle's definition of bending is not a principle proprietary to mathematical kinesiology – as we have seen in this chapter (above, "Bending and Rest"), Aristotle defines bending and employs this definition in explanations of inanimate bodies in *Mete.* IV as well – it is likely that mathematical kinesiology is a branch of a more comprehensive science. A plausible suggestion is that it is a branch of kinematics, which is itself a branch of mechanics. But this proposal is not without its difficulties. First, Monte Johnson defines a *mechanical explanation* as "a demonstration proper to the science of mechanics (primarily of simple machines, such as the screw, lever, pulley, pump, etc.)" and defines a *mechanistic explanation* as "a demonstration directly modeled on a mechanical explanation (for example, a biomechanical explanation)" (Johnson 2017: 127). Aristotle's geometrical argument in *IA* 9 is not modeled on explanations of simple machines and so, strictly speaking, is neither a mechanical nor a mechanistic explanation. On the other hand, arguments that would presumably belong to mathematical kinesiology, such as the argument we discussed concerning how people arise from a sitting position (Figure 7.2), belong to the *Mechanica*. Second, if mathematical kinesiology is ultimately a branch of mechanics it is unclear to which pure mathematical science it is subordinate. Aristotle says that mechanics is subordinate to stereometry (*APo* I 13, 79a1), but elsewhere he says that it is subordinate to geometry (*APo* I 9, 76a24). The arguments Aristotle gives in *IA* 9 employ principles of planar geometry, not solid geometry. But this would be evidence for mathematical kinesiology being a branch of a different applied mathematical science as much as it would be evidence for mechanics being subordinate to geometry. Perhaps Aristotle follows Plato and maintains that stereometry is in some sense posterior to geometry (*Rep*. VII, 528 B–E). Though this would, in a sense, dissolve the issue, whether Aristotle does view geometry as prior (and precisely how he would conceive this priority) is far from clear.

## Other Joints and a Second Geometrical Argument [*IA* 9, 709a8–24]

Aristotle already extends the results of his geometrical argument to animals that walk without bending at the knee when he says that, "since the legs are equal, the one at rest must bend either at the knee or, in any kneeless animal that walks, at some other joint" (709a2–4). In the chapter's next section, he provides several concrete examples of animals that walk in just this way.

> It is, however, possible to move even if the leg has no bend, as when children crawl. (This is the old account of the movement of elephants, but it is untrue.) Such a crawling movement involves a bending in the shoulders or the hips. But nothing could progress upright in this way continuously and safely, but would only move like men in the wrestling schools who convey themselves forward through the dust on their knees. (*IA* 9, 709a8–14)

Aristotle rejects elephants as an example of such locomotion; though their forelimbs bend outward at the knee, they do in fact have knees (Figure 7.7).[18] But the other examples suffice to show such locomotion is possible. What is it though, to bend at the hip?

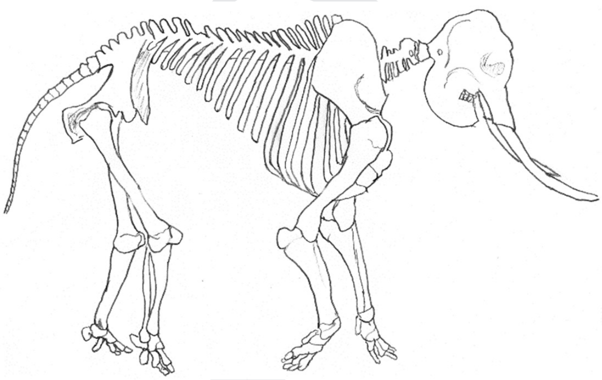


Figure 7.7 The articulation of elephants

All walking requires some bending at the hip, namely, the bending which advances the lead leg forward. But the bending that is relevant to Aristotle's argument would be a bending that effectively increases the length of the lead leg or effectively shortens the length of the trail leg.

[18] At *PA* II 16, 659a29, Aristotle says that elephants' legs bend with difficulty and that their primary purpose is support. Elsewhere, however, he reiterates his claim that they can in fact bend at the knee (*HA* II 1, 498a8–13).

Only then would walking conform to the geometrical constraints that Aristotle has shown govern walking. Neither limb actually lengthens or shortens. But one can bend at the hip in a way that changes the distance between one's feet and the center of one's hip (Figure 7.8).

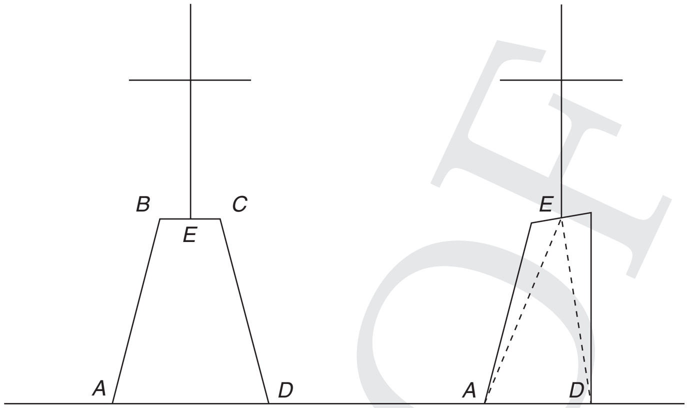


Figure 7.8 Bending at the hip

If one bends both legs at the hip, using the center of one's hip as a fulcrum, the foot of the leg extending from the lower side of the hip (AE) will be farther away from this center point than the foot of the leg extending from the higher side of the hip (DE). So if one bends one's hips in this way, the two changes in length can satisfy Aristotle's geometrical constraints.

At this point, Aristotle returns to the geometry of walking. He provides a second geometrical argument that depicts walking in a way that more closely resembles how we perceive it to occur. But this second argument follows a recapitulation of his first.

> The reason is that the upper portion of the body is big, so the leg must be long; consequently, there must be a bending. Since a standing position is perpendicular <to the ground>, if that which moves forward does not bend, it will either fall as the right angle becomes less, or else it will not advance at all. If one leg is at right angles and the other is advanced, the latter will be at once equal and greater; it will be equal to the leg at rest and also to the hypotenuse of the right-angled triangle. Therefore, that which goes forward must bend, and while bending one, extend the other leg, and incline forward at the same time and make a stride and remain above the perpendicular; for the legs form an isosceles triangle and the head goes lower when it is perpendicular to the triangle's base. (*IA* 9, 709a14–24)

In this discussion, the walker begins not by advancing their lead leg, but by leaning forward (Figure 7.9*b*). If someone standing upright begins to lean forward, their center of gravity will no longer be over either of their feet. If they lean forward too far, they will fall onto their face.

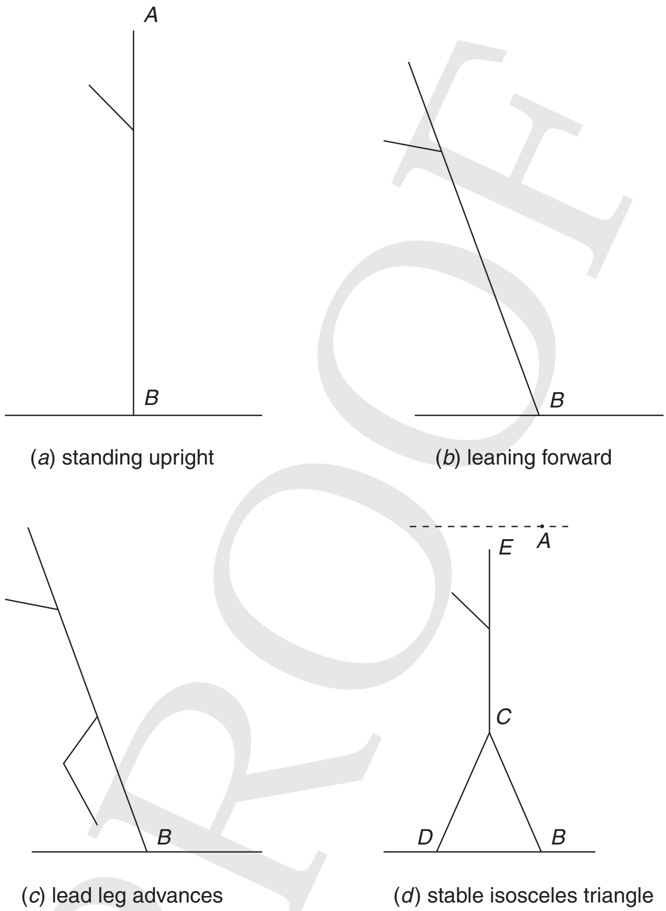



Figure 7.9 Moving forward with bending

In order to prevent falling over, the walker must advance one leg and extend the trail leg, presumably by bending at the ankle, in order to continue forward momentum and facilitate the placement of the lead foot on the ground (Figure 7.9*c*). Aristotle does not say why the lead leg must bend. It is not required by his geometrical constraints. But it is required in order to overcome the friction between the lead foot and the ground.

Without bending at the knee or hip, moving one's leg forward can only be accomplished with great effort (Figure 7.10*a*). At a minimum, one must shift one's weight to the trail leg. But such shifts of weight involve bending at the hip. The need to bend the lead leg is only exacerbated if one is leaning forward. For in this inclined position, moving one's lead leg without bending it would initially move it downward into the ground (Figure 7.10*b*).

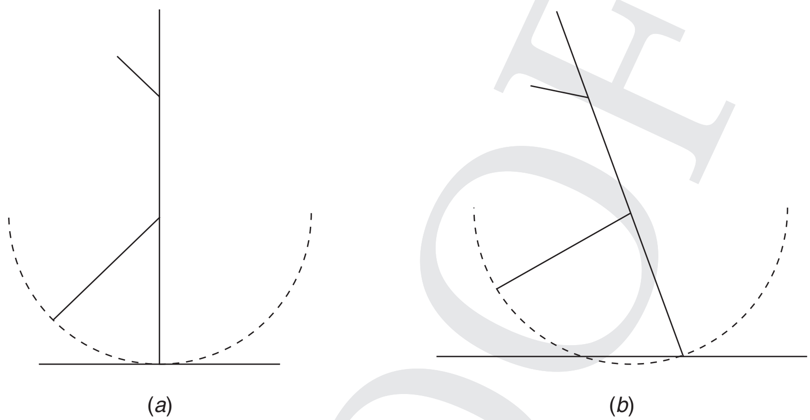



Figure 7.10 Moving forward without bending

At midstride, with one's lead leg now on the ground, the legs form an isosceles triangle (Figure 7.9*d*). The walker's weight is not above either of the feet. But a line can be drawn from the head to the midpoint of the isosceles triangle's base that is perpendicular to the ground, and having one's center of gravity midway between one's feet is a stable position. This depiction also conforms to the observable phenomena to which Aristotle appeals in support of his first geometrical argument. For the walker's head will be at a lower position when his legs form an isosceles triangle than it is when he stands upright.

## Other Varieties of Animal Locomotion [*IA* 9, 709a24–b19]

Aristotle concludes the chapter by extending the result that walking requires bending to other varieties of animal locomotion. He begins with the locomotion of footless animals.

> Some footless animals advance by undulations (this happens in two ways: for some, the bending is upon the ground, such as with snakes, while for

> others it is up and down, such as with caterpillars) and undulation is bending. Others move by oozing, such as what are called earthworms and leeches. (*IA* 9, 709a24–29)

The three cases he discusses are (i) undulations in which what bends remains on the ground, (ii) undulations in which the bending lifts part of the animal off of the ground, and (iii) oozing (ἰλύσπασις).[19]

This last variety of locomotion is curious. Aristotle says, "For these advance with one part leading the way, and then drawing the remainder of the body to them, and in this way they change from place to place" (709a29–31). Both earthworms and leeches are segmented, and this has led many to model this form of locomotion on either telescopic extension, in which the segments would overlap when the body is drawn together, or on concertina-like extension and compression.[20]

Neither of these suggestions is factually correct. We now know that earthworms move by means of peristalsis, that is, by alternating between (i) radially symmetrical contractions of circular muscles that wrap around each of their body's segments, and (ii) contractions of longitudinal muscles that extend lengthwise through their cylindrical hydrostatic bodies. The worms first anchor their body's posterior to the ground with small hair-like bristles called setae and contract the circular muscles located in their body's anterior (Figure 7.11*a* and *b*). These contractions compress the anterior segments thereby pushing the fluid in their hydrostatic skeleton forward and elongating their anterior. The earthworm then anchors its body's anterior to the ground and contracts its anterior's longitudinal muscles (Figure 7.11*c*). The contraction of their longitudinal muscles reverses the process and makes their body's anterior thicker and shorter thereby drawing the central segments of its body forward. By cascading these contractions along the length of their bodies, earthworms are able to progress anterogradely (Figures 7.11*d* and *e*).

Though neither the contraction of circular muscles nor the contraction of longitudinal muscles involves bending as Aristotle defines it, there is at least one respect in which this is immaterial to our interpretation. For Aristotle would have rejected the contemporary account of earthworm



[19] The neologism ἰλύσπασις is derived from ἰλύς which means mud or slime.

[20] Though I ultimately disagree, Farquharson 1912 at least makes a reasonable choice when he translates ἰλύσπασις as "telescopic action" and notes the parallel to concertinas. Farquharson rightly criticizes the LS translation "wriggling," "which is not a worm's normal movement." For the same reason, I maintain that the LSJ translation, "crawling," is incorrect, though both Forster 1937 and Louis 1973 follow this recommendation.

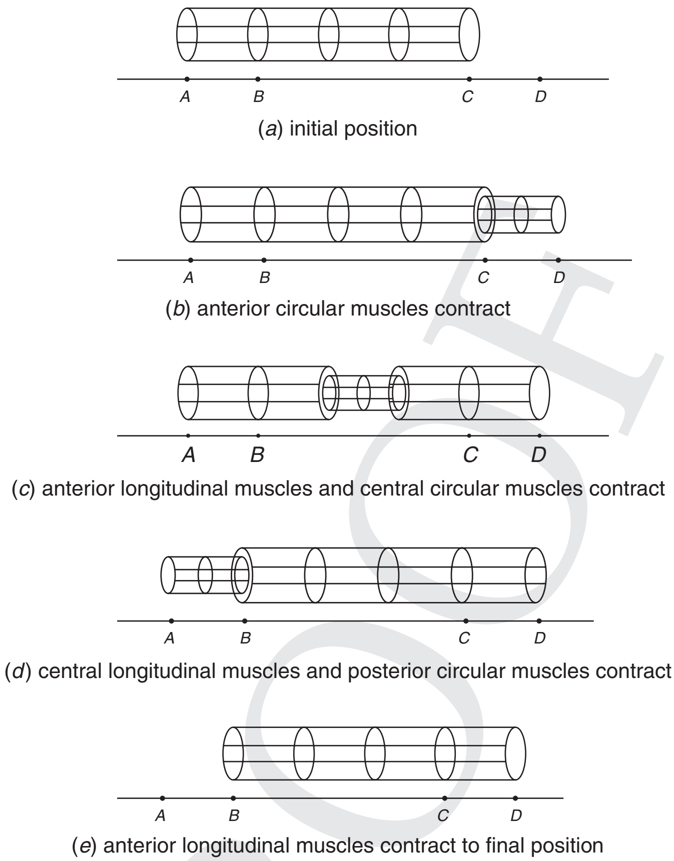



Figure 7.11 Diagrammatic representation of worm movement

locomotion altogether because he infamously fails to appreciate that muscles play *any* significant role in bodily movement and instead attributes this role to sinews (νεῦρα).[21] Nevertheless, this case remains problematic even on Aristotle's mistaken view. First, sinews effect movement "through contracting and relaxing," so their operation is not unlike that of muscular contraction.[22] Second, and more important, if Aristotle's

[21] Aristotle thought that muscles were a type of soft flesh with two main functions: (i) protecting other parts of the body, and (ii) being a medium for the sense of touch. For a thorough and convincing discussion of Aristotle's views on muscles and sinews, see Gregoric–Kuhar 2014.

[22] *PA* III 4, 666b14–15: διὰ τοῦ ἕλκειν καὶ ἀνιέναι (cf. *MA* 7, 701b9–10).

understanding of oozing conforms even minimally to what is observable, then it will primarily involve the sides of the worm's body becoming concave and then straightening.[23] But each of these changes also involves a change in the body's length. So even on Aristotle's preferred account, oozing seems to be a case in which locomotion occurs principally by parts of the body changing their size or length, and this prevents these bodily changes from being instances of bending. Consequently, this appears to be an example of animal locomotion that does not involve, let alone require, bending.[24]

The two varieties of undulation Aristotle describes are not saddled with the same difficulties; they both require bending. For,

> it is clear that, if the two lines they form were not greater than the one, movement would be impossible for undulating animals. The reason is that, when the bend is extended, they would not have made any advance, if it subtended an equal line; as it actually is, the line is longer when it is extended, and then this part stays still and draws up the remainder. In all the aforementioned changes, that which moves advances by first extending itself straight and then by curving itself; it straightens itself with its leading part and curves itself in the parts which follow. (*IA* 9, 709a31–b7)

The two lines Aristotle mentions are the lines between the endpoints of the curve that undulating animals form and the vertex of this curve (Figure 7.12). Aristotle notes what should be obvious: the length of the sum of these lines, AB + BC, is greater than the length of the straight line between the curve's endpoints, AC. One could interpret Aristotle as making a more substantive claim, namely, that *each* of the lines AB and BC, not their sum, must be greater than AC. On this second reading, if undulating animals are to progress, the angle or curve they produce must be particularly acute. But this is not necessitated by any geometrical



[23] Though unlikely to have been written by Aristotle, the *Problemata* provides some evidence that Aristotle would countenance these changes in the sides of worms being instances of bending insofar as it asserts that hot water is a cause of wrinkles and describes wrinkles as a bending of the skin (*Prob.* XXIV 7, 936b10–12).

[24] This kind of locomotion does, however, require that there be at least one fixed point at rest. Setae fix the worm's posterior when the anterior contracts and advances and similarly fix the worm's anterior when the worm's posterior contracts and is brought forward to meet the rest of the body. So without some part of the body being at rest, earthworms cannot progress. Though Aristotle often allows for exceptions to his principles, especially in the biological works, this example is problematic insofar as (a) Aristotle can't abandon his definition of bending without significantly undermining his previous arguments, and (b) the placement, without comment, of oozing among other varieties of animal locomotion he explicitly maintains involve bending makes it unlikely that he considers it exceptional.

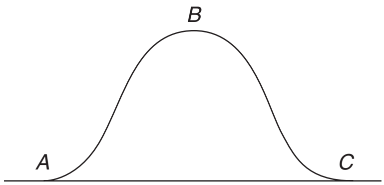



Figure 7.12 Geometrical properties of undulation

considerations, nor is it needed to show that undulation is bending, and Aristotle fails to supply any reason for thinking that undulation must (or even does) occur in this way.

When an undulator's body bends, the anterior end point remains fixed and draws the posterior toward it. When the body straightens, the posterior end point remains fixed and the anterior is extended forward.

Aristotle's final remarks concern animals that jump, fly, and swim.

> All jumping animals as well make a bend in the lower part of the body, and jump in this manner. So too flying and swimming animals progress, the one straightening and then bending their wings to fly, the other their fins to swim. Some of the latter have four fins and others, those with a longer shape, for instance eels, have two. These move by substituting a bending of the rest of their body for the missing pair of fins, as we have already said. Flat fish use two fins, and the flat part of their body instead of the second pair of fins. Really flat fish, like the ray, produce their swimming with the actual fins and with the outer periphery of their body, alternately bending and straightening. (*IA* 9, 709b7–19)[25]

According to Aristotle, the locomotion of birds and fish is largely analogous. Just as birds bend their wings to fly, so fish bend their fins to swim. And just as birds use their tails like the rudder of a ship to direct their flight (cf. *IA* 10, 710a1–4), so fish use their tails to direct their swimming.

[25] Aristotle makes similar divisions among swimmers in the *HA*: "Of swimming creatures that have no feet, some have fins, as fishes: and of these some have four fins, two above on the back, two below on the belly, as the gilt-head and the bass; some have two only – to wit, such as are exceedingly long and smooth, as the eel and the conger; some have none at all, as the muraena and others that use the sea just as snakes use the dry ground – and snakes swim in water in just the same way. Of the selachia some have no fins, such as those that are flat and long-tailed, as the ray and the sting-ray, but these fishes swim by means of their flat bodies" (*HA* I 5, 489b24–32, trans. Thompson; cf. *PA* IV 13). On bending in birds, see also *HA* II 1, 498a29–31. Note that there are no explanations offered in this passage; it comprises only descriptions of swimmers' features and points to some of the ways in which these features are correlated. It is, consequently, an example of observational science whose purpose is to know the fact (τὸ μὲν ὅτι) rather than to know the reason why (τὸ δὲ διότι).

As a matter of fact, the way most fish swim is exactly the opposite of what this second analogy claims. Movement in the tail causes the forward momentum of most fish and the principal function of the pectoral and ventral fins is to direct this motion. But despite these inaccuracies, jumping, flying, and swimming conform to Aristotle's thesis that animal locomotion requires bending.